

1- Summary

$$\Delta\phi = \Delta\psi + N \cdot 2\pi$$

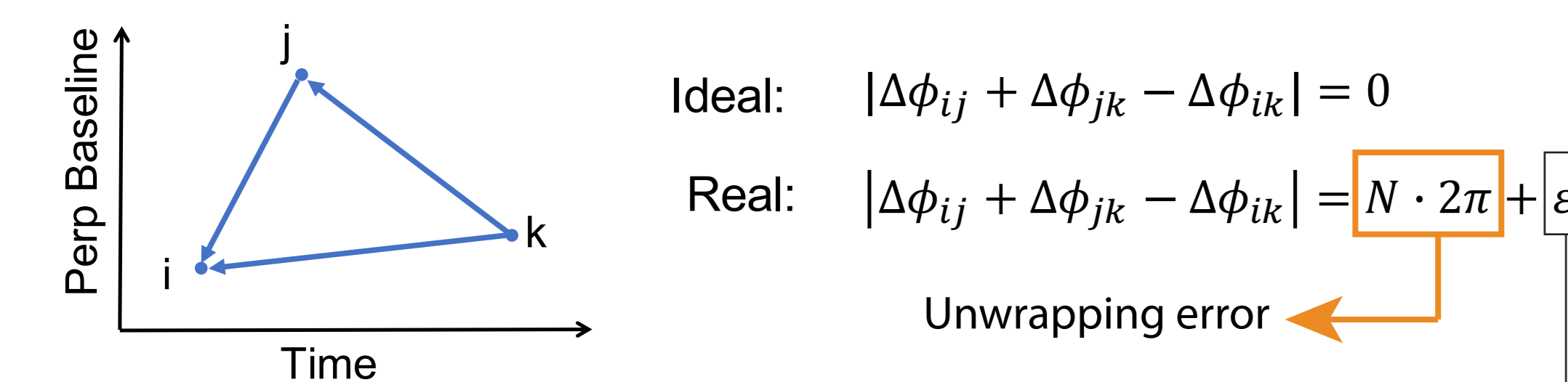
We examine the **characteristics** of unwrapping errors in InSAR time-series analysis.

We present **two methods** for unwrapping error correction for interferograms stack.

We show a preliminary model for a hybrid method of the two methods above.

We provide **strategies to suppress** the influence of residual unwrapping errors in InSAR time-series analysis.

2 - Characteristics of Unwrapping Errors



Processing inconsistency: filtering, coregistration and interpolation.

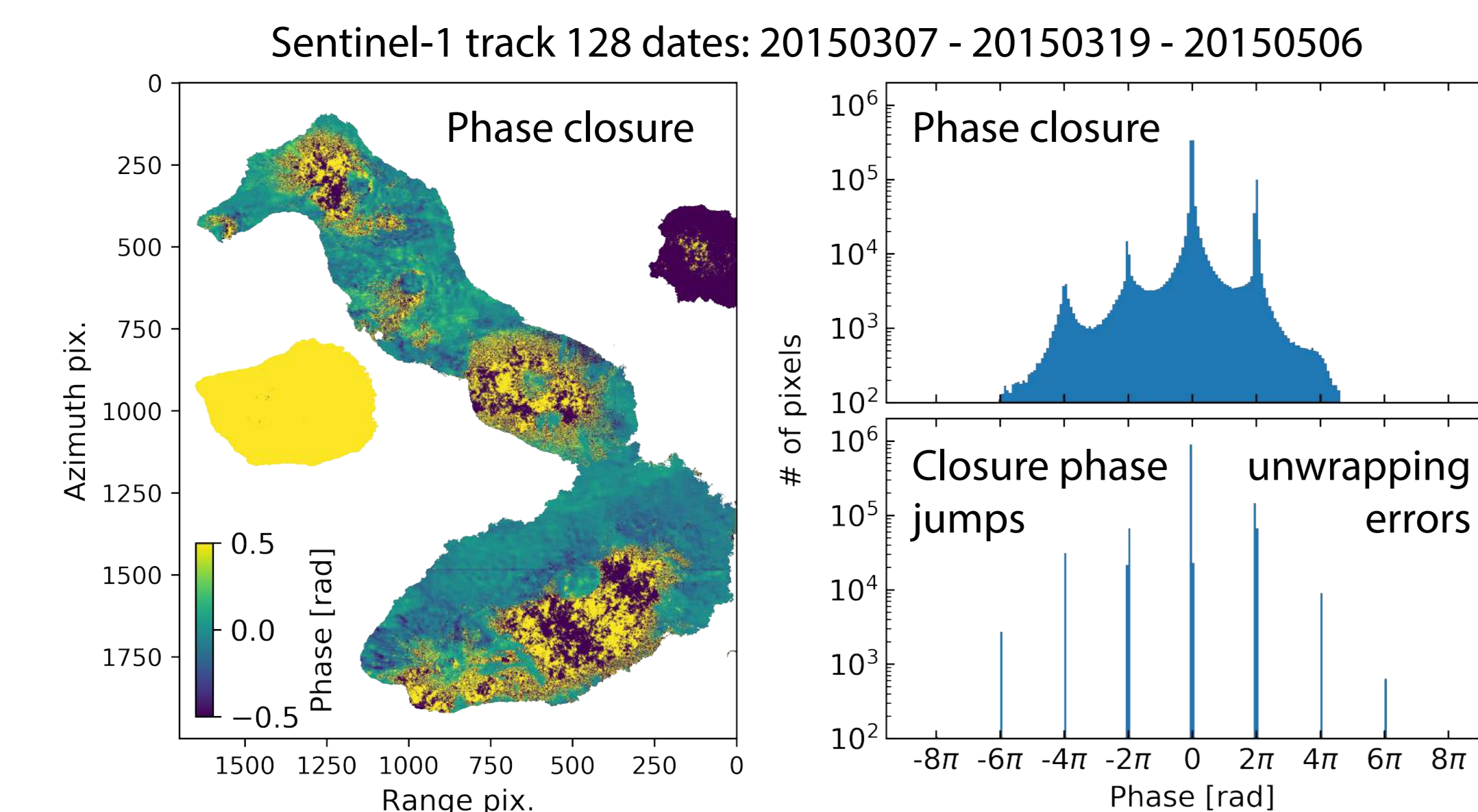
Dielectric properties change such as soil moisture

Decorrelation noise: geometrical, temporal, thermal

• Non-zero phase closure jumps of one interferograms triplet

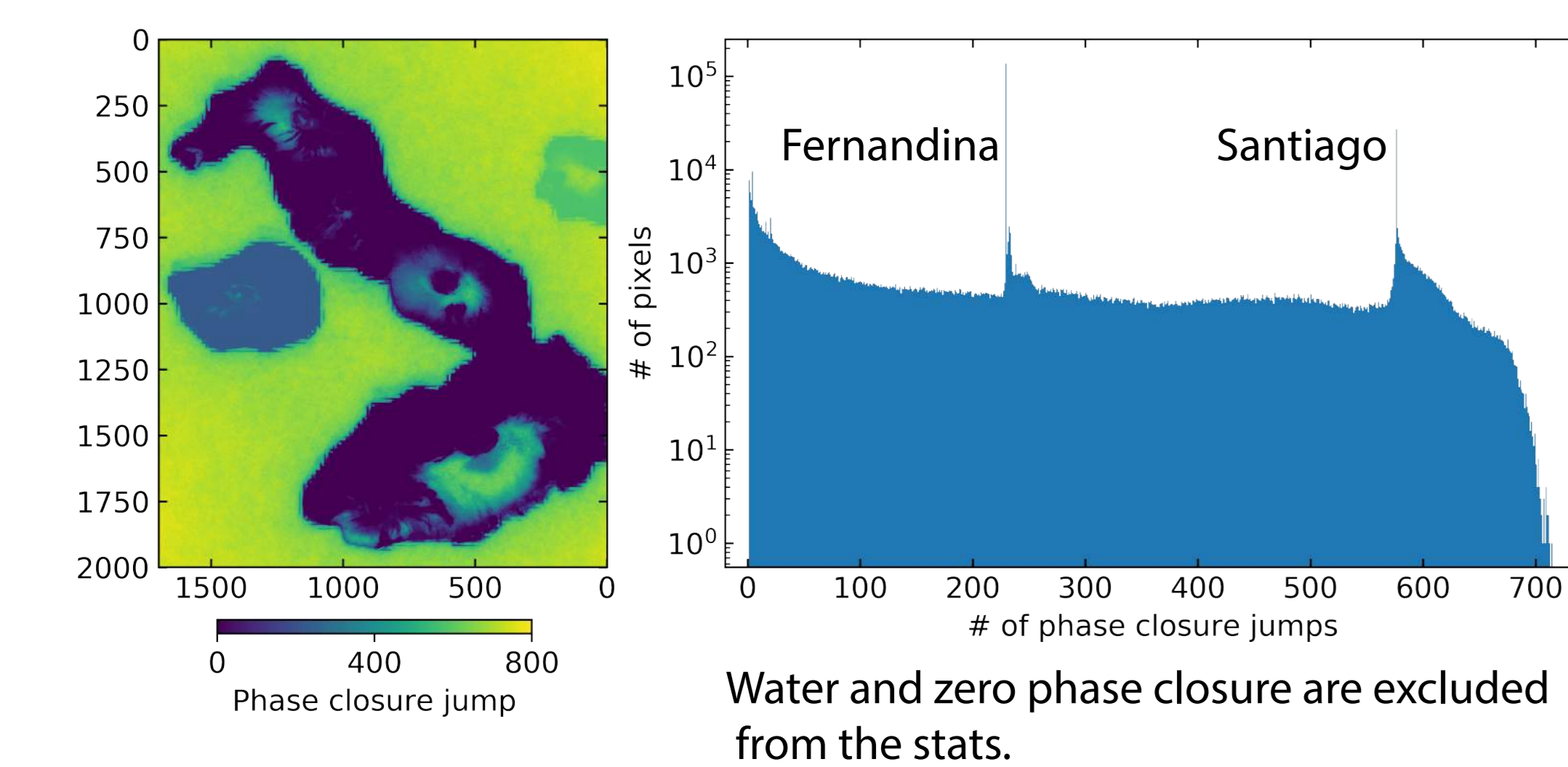
Phase closure: $C_{ijk} = \Delta\phi_{ij} + \Delta\phi_{jk} - \Delta\phi_{ik}$

Phase closure jump: $C_{ijk}^{jump} = C_{ijk} - wrap(C_{ijk})$



Unwrapping errors -> the **integer component of non-zero phase closure, or phase closure jump.**

• Distribution of non-zero phase closure jumps



Reliable regions: Fernandina and Santiago islands.

References

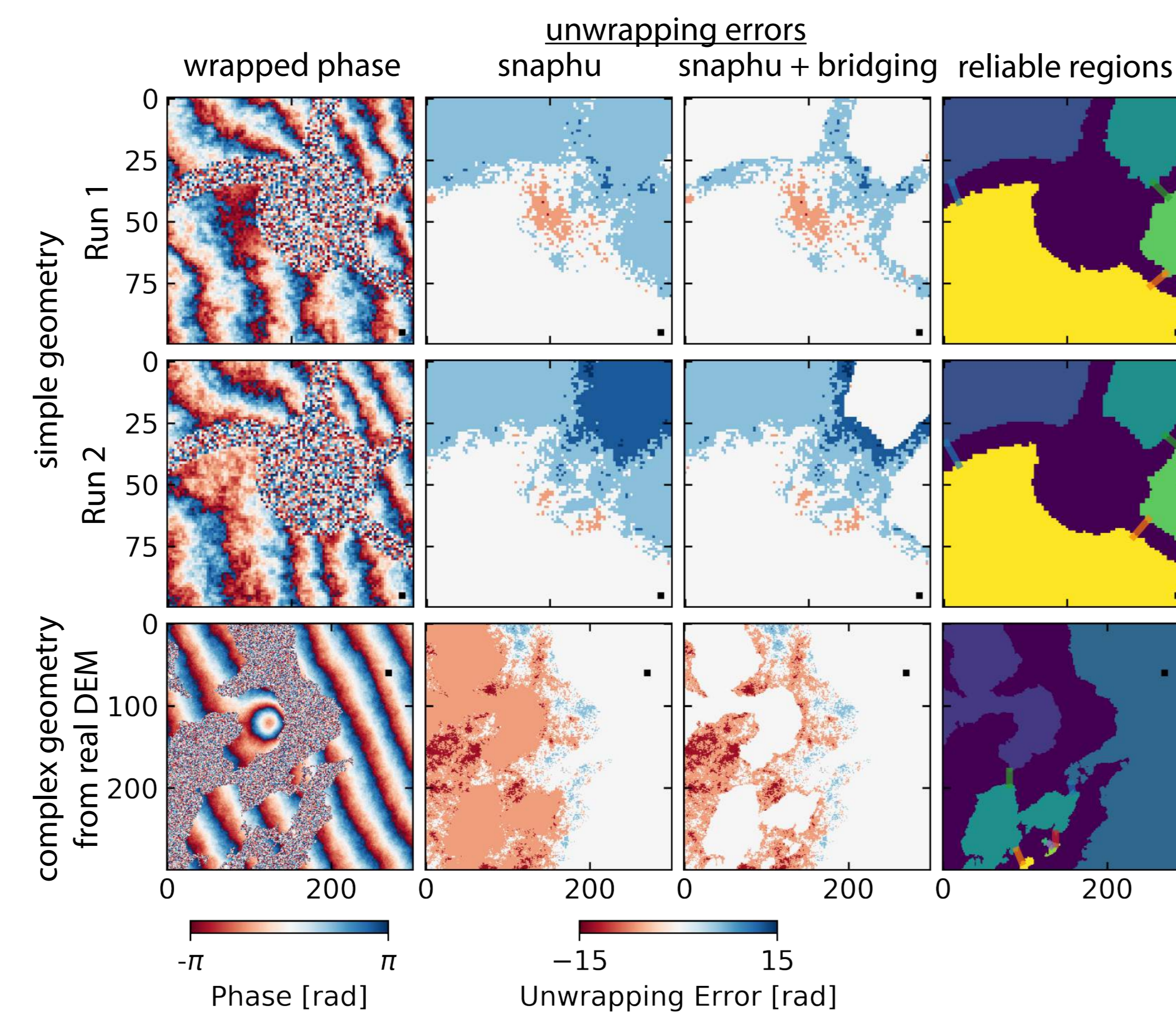
• Yunjun Z., H. Fattahi, F. Amelung, PySAR - An Efficient Framework for Time Series Analysis of Small Baseline InSAR Stack (in prep).

• Fattahi, H. (2015), Geodetic Imaging of Tectonic Deformation with InSAR, 190 pp, University of Miami, Miami, FL.

Code is available at Github: <https://github.com/yunjunz/PySAR>

3 - Bridging of Reliable Regions

• Simulations for one interferogram
 unwrapped phase = defo (mogi) + tropo + ramp + decorrelation noise
 decorrelation noise corresponding to coherence of 0.6 and 0.001



Assumption: phase gradients among reliable regions (connected components) are less than one-half cycle (π rad).

To fulfill the assumption of smooth phase over reliable regions, one could remove contributions from troposphere, DEM error, deformation model, ramps etc before phase unwrapping, and add them back after unwrapping error correction.

Procedure:

1. Identify reliable regions using the connected components from SNAPHU (Chen and Zebker, 2001). Discard small regions and pixels on the boundary.

2. Construct directed bridges to connect all reliable regions: **MST** network of bridges with min total distance + Breadth-first ordering.

3. Estimate the integer numbers of phase offset between two regions connected by each bridge.

- 1) Estimate a linear or quadratic ramp based on the largest region and remove it from all pixels.
- 2) Use median value within a window centered at the endpoint of the bridge, instead of the single pixel, to calculate the phase difference.

Suitable for: phase unwrapping errors occurred on areas separated by narrow decorrelated features such as **narrow water bodies** or **steep topography**. Not suitable for unwrapping errors caused by high spatial phase gradient such as Sierra Negra, Galapagos.

Limitation: The MST bridges is sensitive to bridge locations and the fringe patterns, could be improved by more redundant bridges construction method.

4 - Phase Closure of Interferograms Triplets

$$\begin{bmatrix} A & L \\ -2\pi C & D \\ \alpha I \end{bmatrix} U = \begin{bmatrix} C\Delta\phi - wrap(C\Delta\phi) \\ 0 \\ 0 \end{bmatrix}$$

D is an extra constraint for interferograms "free of unwrapping errors": all triplets associated with this interferogram have zero phase closure jump. (Fig on the right: all interferograms except for 4, 6 and 8)

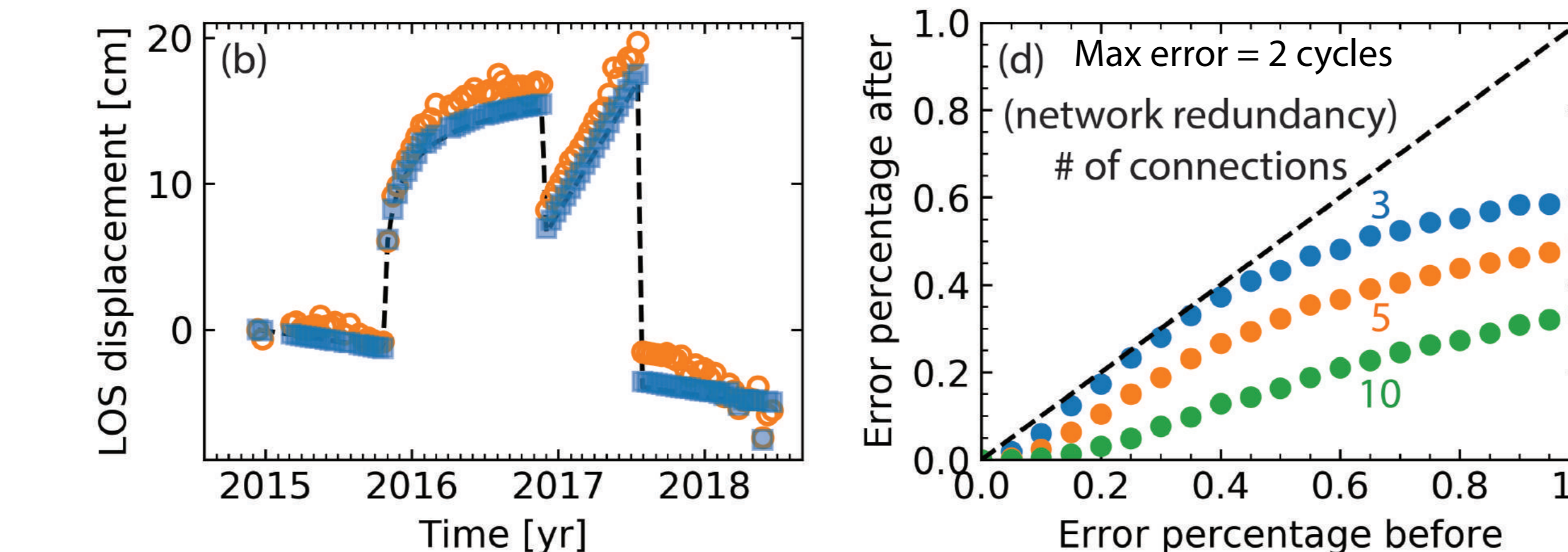
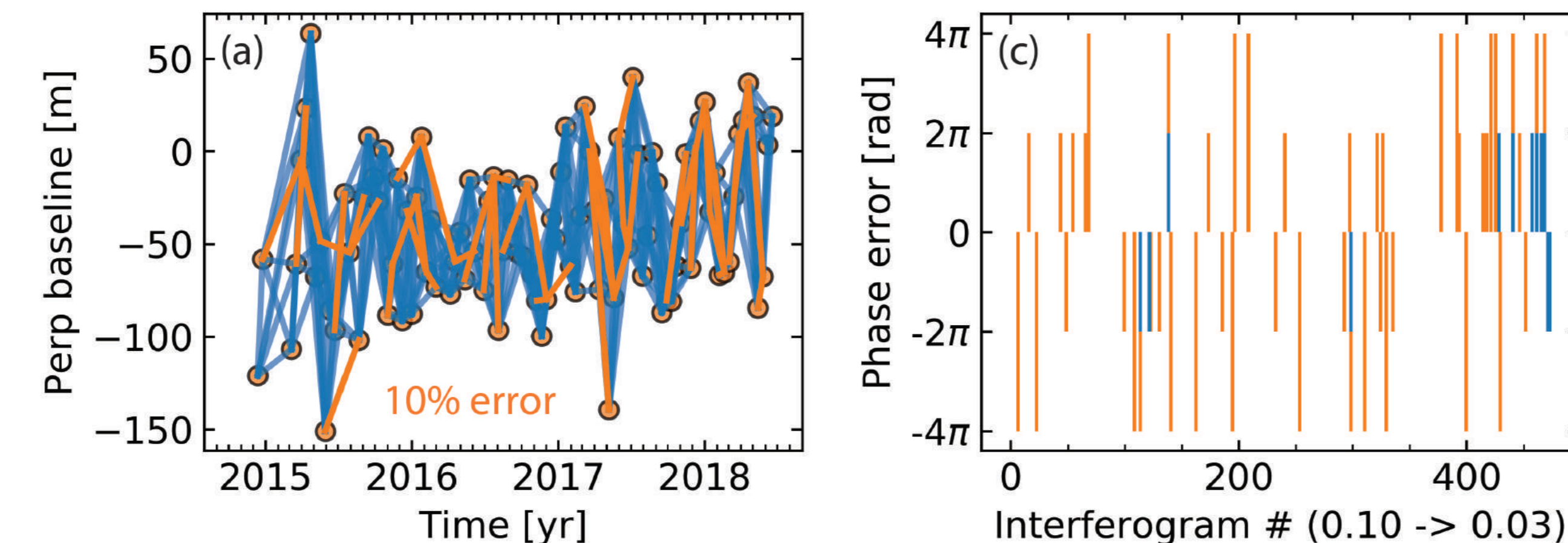
αI for Tikhonov regularization when network is not redundant (underdetermined).

$$\hat{U} = round((A^T A)^{-1} A^T L)$$

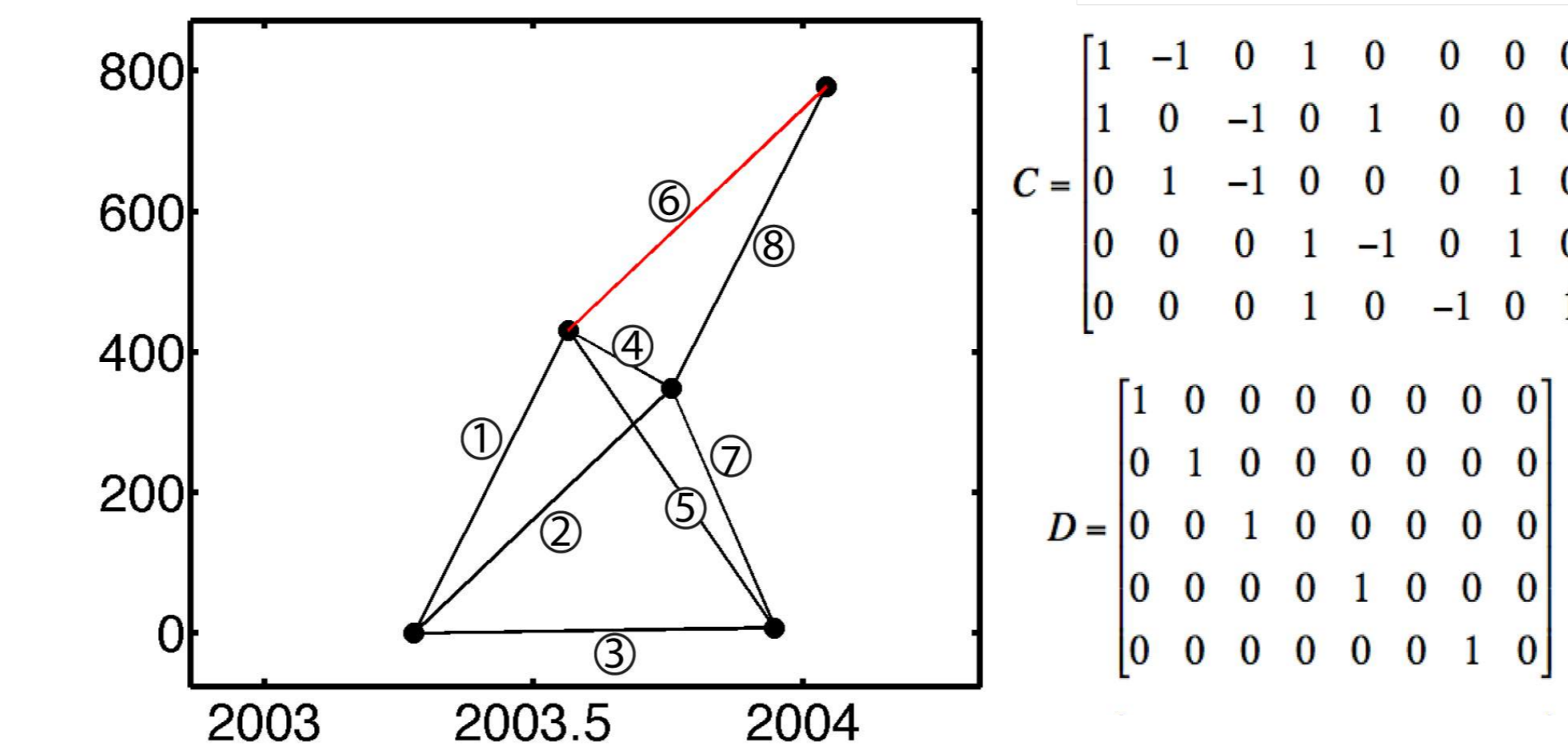
$$\Delta\phi^c = \Delta\phi + 2\pi \hat{U}$$

• Simulations for a interferogram stack on one pixel

1. Sequential network with 5 connections based on real Sentinel-1 temp/spatial baseline.
2. Randomly select 10% interferogram to add unwrapping error with magnitude randomly selected from 1 to 2 cycles.
3. Compare the percentage of interferograms with errors before and after correction.
4. Repeat step 2-3 with different input error percentages.
5. Repeat step 1-4 with different numbers of sequential connections.



• Examples for design matrix C and D (Fattahi, 2015)

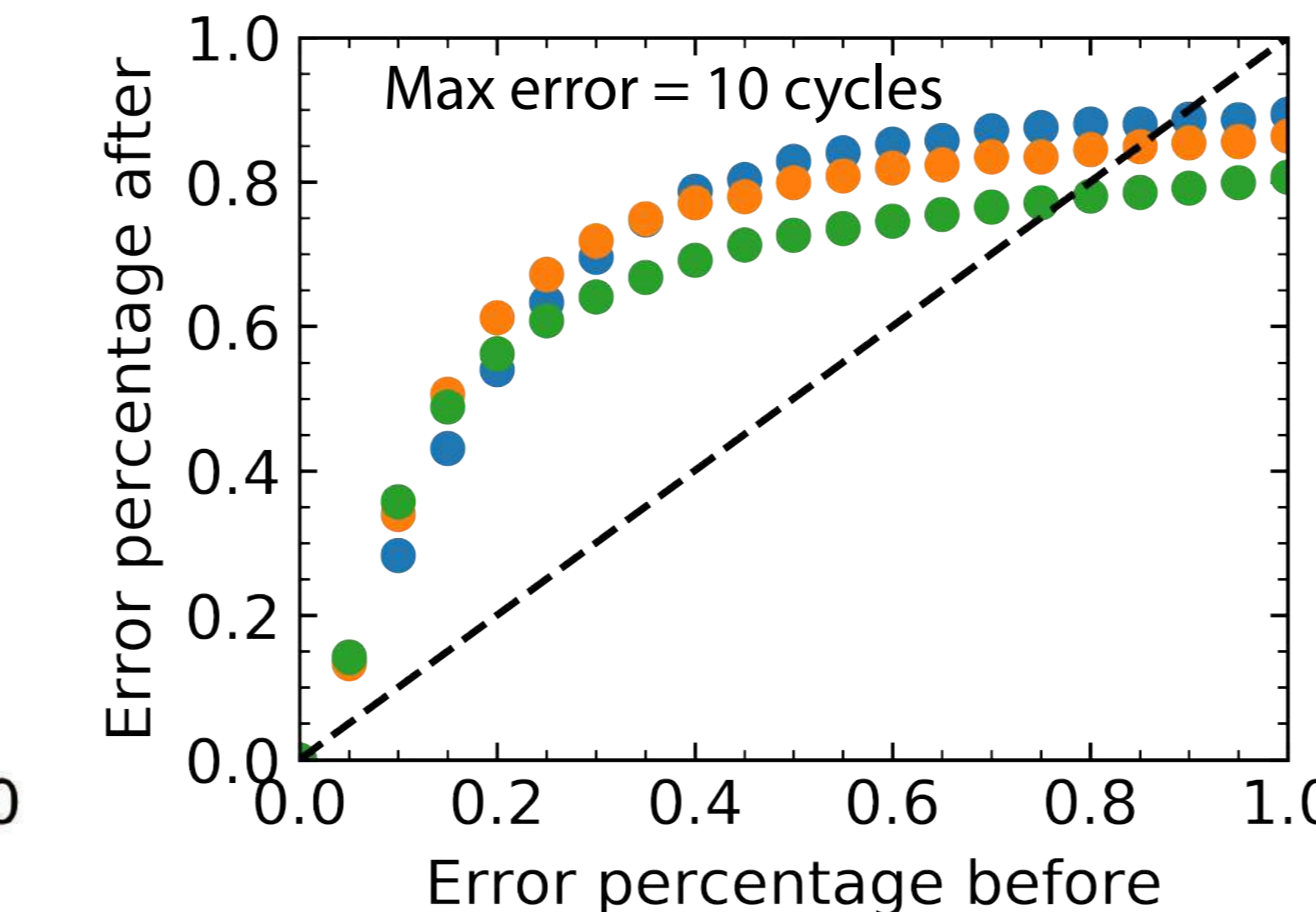


(b) -> unwrapping errors introduced **bias in the estimated time-series displacement**, and should be corrected for in order to achieve reliable InSAR time-series measurement.

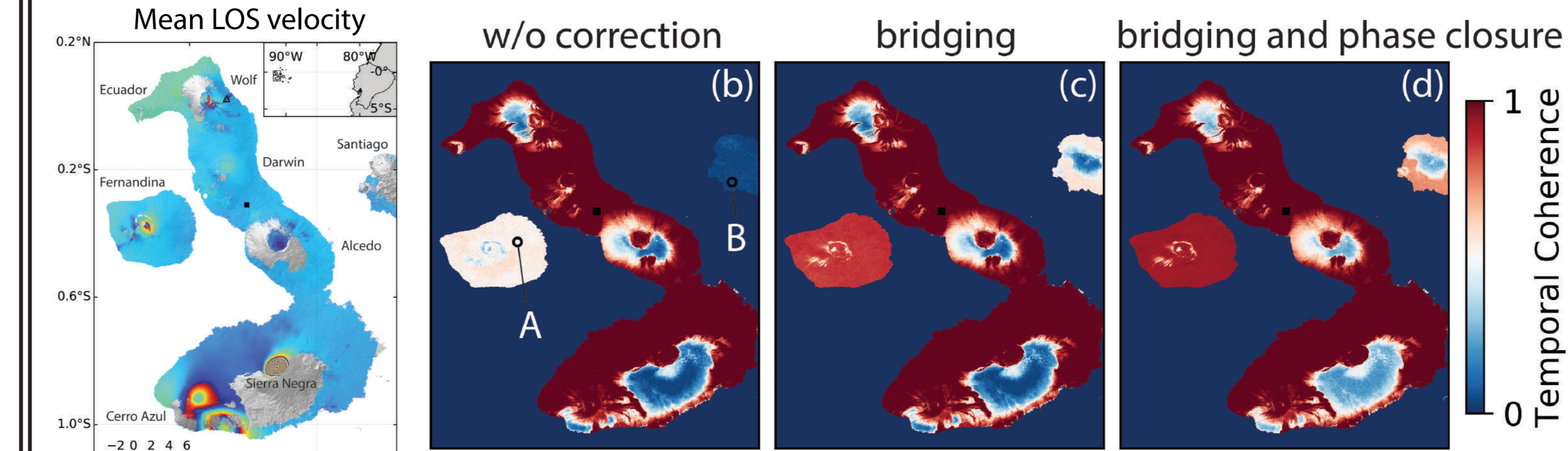
(c) -> phase closure method could significantly reduce unwrapping errors; but not all of them, especially when the network redundancy is not high or a lot of unwrapping errors represents.

Suitable for:

- 1) unwrapping errors with **small magnitude**
- 2) **redundant network** (more sequential connections) number of triplets (observations) increases faster than number of interferograms



5 - Application to Galapagos with Sentinel-1 data



The bridging assumption is not restrict at all in this case, but we apply it anyway. No indication shows localized deformation between Isabela and Fernandina / Santiago.

Data: Sentinel-1 desc track 128
 Time: Dec 2014 - Jun 2018 (98 images)
 Software: ISCE-topsStack + PySAR

We used SNAPHU for phase unwrapping; then applied **bridging method followed by the phase closure method**, because these two methods are independent.

Temporal coherence
 Pixel A on Fernandina: 0.49 -> 0.86 -> 0.93
 Pixel B on Santiago : 0.07 -> 0.57 -> 0.71

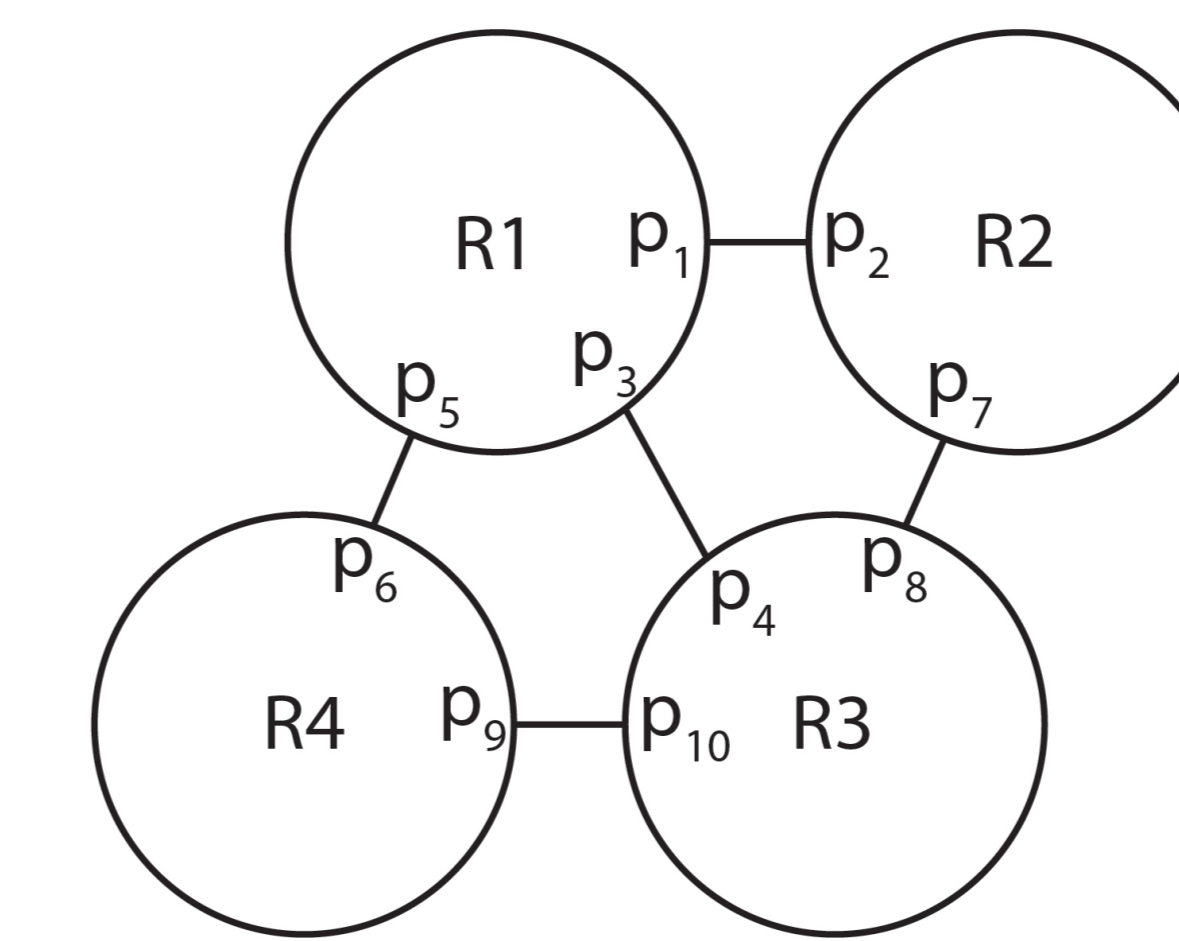
6 - Joint Correction with Bridging and Phase Closure (conceptual)

With reliable regions from 2D phase unwrapping algorithm, the 3D unwrapping error correction can be transformed into the following task:

Determine the integer-cycle phase offsets to be added to each points of bridges for each interferogram ($M * P$ unknown integers).

M = number of interferograms

P = number of points used to construct bridges in each interferogram



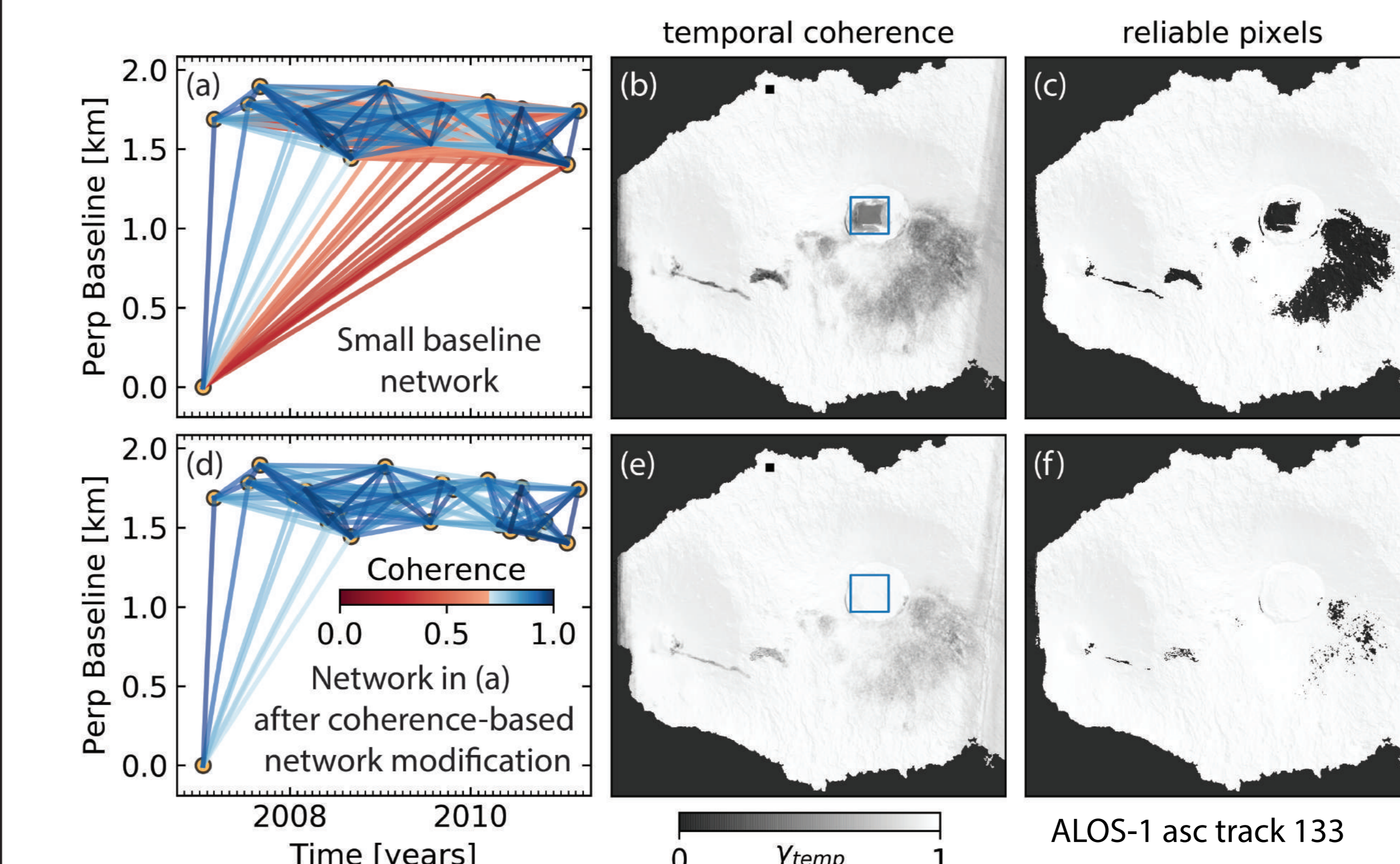
Constraints:

- 1) phase difference between two endpoints of each bridge in each interferogram is less than π in magnitude (Box 3; weak constraint by assigning low weight)
- 2) phase offsets among points inside the same reliable regions in each interferogram is the same.
- 3) each point has a phase closure constraint in time domain: zero phase closure jump of each interferogram triplet (Box 4)

Global minimum problem in 3 dimension (after 2D phase unwrapping for each interferogram)

Similar to 3D phase unwrapping on sparse points as Agram and Zebker (2010) and Costantini et al. (2012), but different on constraint 2 and 3.

7 - Strategies for handling residual unwrapping errors in InSAR time series



1. Weighted network inversion (phase linking).

-> suppress the influence of outliers (residual unwrapping errors and decorrelation noise) on low coherent pixels.

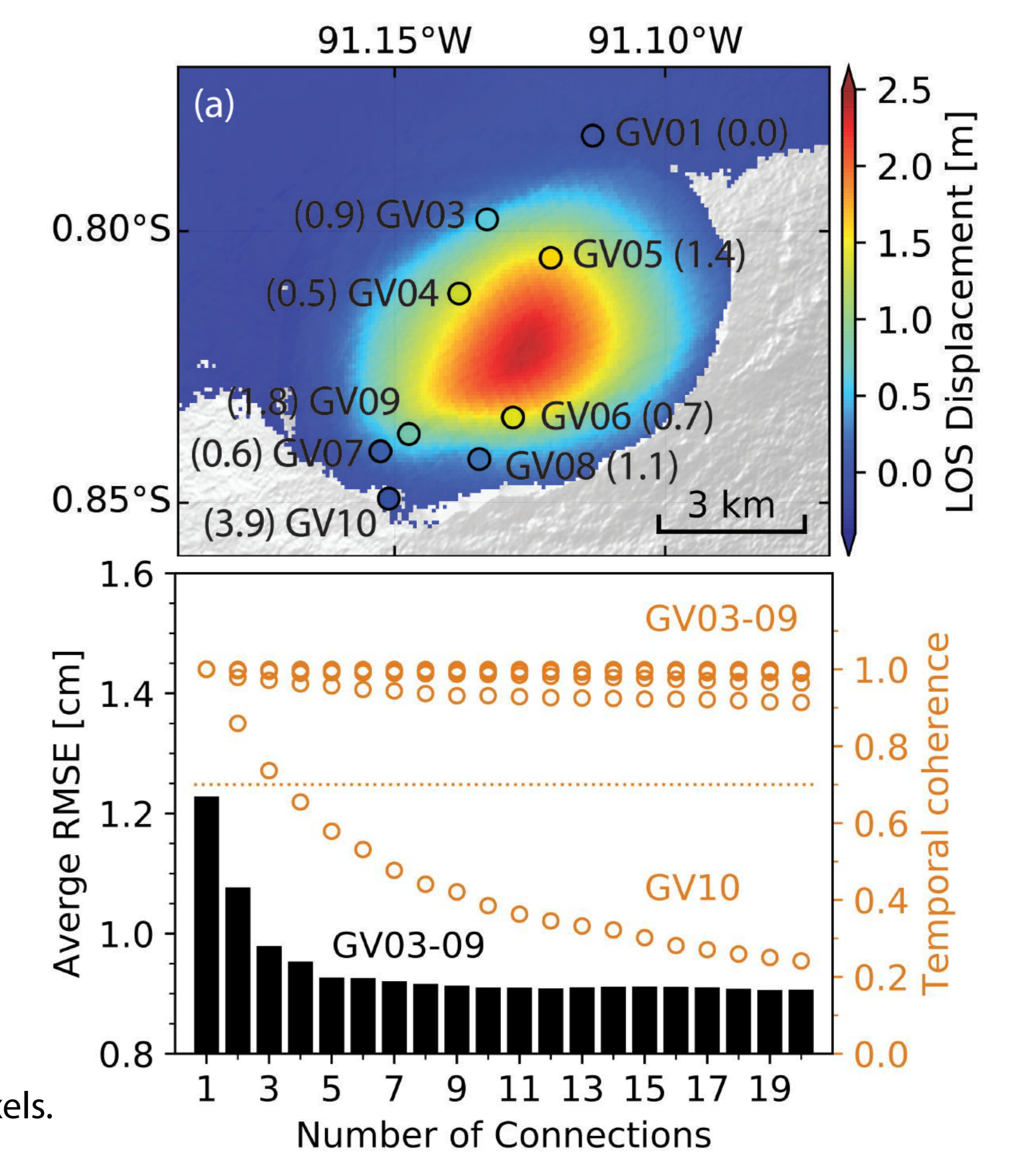
2. Coherence-based network modification.

-> exclude interferograms with residual unwrapping errors on moderate or high coherence pixels

Based on the empirical observation that reliable regions with unwrapping errors are usually surrounded by decorrelated areas. Thus a customized area of interest including those decorrelated areas would leads to a low average spatial coherence.

3. Generate more potentially coherent interferograms with a relatively relaxed interferogram selection thresholds.

- i.e.: more connections in sequential network; larger temporal and spatial baselines in small baseline network for Sentinel-1, use 5 connections (98 images -> 475 interferograms)
- > provide room for network modification
 - > keep redundancy -> better unwrapping error correction with phase closure
 - > more observation and less affected by decorrelation noise.
 - > more robust estimation of temporal coherence as reliability measure of InSAR time-series



• Assess the impact of network redundancy on the estimation:
 1) the displacement time-series and
 2) the temporal coherence (the reliability measure)
 by considering stacks of Sentinel-1 interferograms with different numbers of sequential connections for each acquisition, and compare the result with GPS.

The average RSME decreases (improves) with increasing number of connections rapidly until 5 connections, then slowly until decrease become negligible.

Temporal coherence stays at high value for all stations except for GV10, which is a un-reliable pixel on vegetated area -> improved identification of reliable pixels.